ELECTROMAGNETIC DETECTION AND IDENTIFICATION OF COMPLEX STRUCTURES

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ABSTRACT

Using electromagnetic waves to detect and identify complex structures near the earth's surface remains challenging because of ground conductivity and ground clutter. Powerful computer-based numerical techniques are required to solve canonical propagation problems involving electric dipoles, magnetic current loops, perfectly conducting and dielectric spheres, cylinders, etc. A critical need remains for real-time realistic and robust analytical techniques that extend detection and identification domains to complex structures. Our study addresses this need using electromagnetic reciprocity in combination with dyadic Green's functions and asymptotic theory of integration.

1. INTRODUCTION

In the past decade theoretical interest for detecting and identifying surface and shallow buried complex targets has been minimal to modest in favor of more expedient experimental studies. As recently reported at the international EUROEM conference (Lausanne, Switzerland July 2008) success in meeting desired useful operating curves for mine detection has not been entirely successful using various combined theoretical and experimental schemes for determining the Probability-of-Detection, P_d , and Probability-of-False Alarm, P_{fa} .

Two reasons given for the slow lack of progress in determining P_d and P_{fa} are: (1) this is a fundamentally difficult detection problem with true "natures limits" on P_d and P_{fa}, and (2) we have pushed our theoretical capability to its limits. It is our perception rather that: (1) we haven't even come close to exhausting detection algorithms, and (2) the powerful technique of dyadic Green's functions (Tai, 1994) has not yet been applied to detection near the ground-air interface. In this paper we address the latter idea. In Section 2 we define the problem to be solved. The theoretical model based on the dyadic Green's function approach is described in Section 3. Detection theory using the integrated concepts of reciprocity (Monteath, 1973) and the asymptotic method of integration (Chew, 1990) are rendered in Section 4. A calculational approach and sample problem are discussed in Section 5.

2. STATEMENT OF PROBLEM AND OVERVIEW

Figure 1 depicts the detection and identification of a buried object. In a related paper von Laven et al. (2008) have presented experimental results dealing with this subject. The physical basis for doing this began nearly one hundred years ago with the observation that the ground played a major role in the propagation of radio waves. Arnold Sommerfeld (1909) provided an elegant theoretical model of how ground conductivity affects the propagation equations. These equations turned out to be so challenging that special mathematical approximations were developed for the case when antennas are located on the earth's surface. In the mid-1930s Norton (1937) developed robust and yet practical models of electromagnetic radiation from horizontal and vertical electric and magnetic dipoles near the earth's surface.

During WWII and moving into the early 1970s research progressed on buried antennas of various sorts and on underwater communications. Ground penetrating radar (GPR) surfaced in the 1970s when utilities tired of continuously repairing underground water lines, gas lines, and electric power cables. GPR is generally successful in

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finding these relatively large targets. In the early 1990s and continuing today there is great concern for detecting large buried objects such as bunkers deep into the ground, but this is different in quantitative ways from detecting the surface and shallow buried objects of interest in this study.

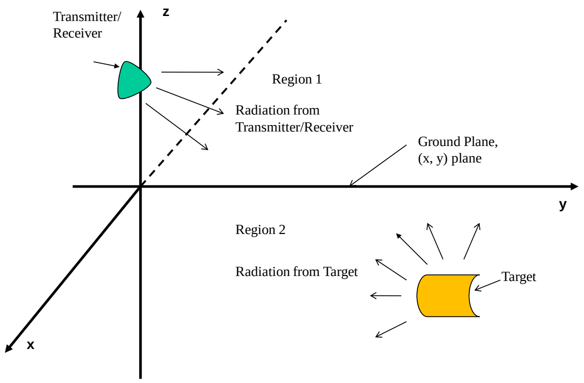

Figure 1. Detecting and identifying buried object.

Our targets differ significantly from canonical electric and magnetic dipoles; detection and identification at suitable range is also critical. Kraichman (1970) has presented a comprehensive treatment of buried dipoles, and Jordan & Balmain (1968) have presented a simplified version of propagation models that include practical curves of attenuation over ground.

The detection process comprises the following steps: (1) generating the below-ground electromagnetic field from the transmitter, (2) generating a below-ground current via target interaction, and (3) determining the aboveground electric field from the below-ground current. In most cases of practical interest, estimating the below-ground field from an aboveground transmitter operating in the 100 MHz and above frequency range is relatively easy, requiring mainly knowledge of the ground reflection coefficients. Predicting the generation of intrinsic target current and related target induced ground current is difficult because detailed knowledge of targets is not always available.

We select a generic target class characterized by the constraint: $\iiint \vec{J}_2(\vec{r}')dV' = 0$. This equation means that the current is generated wholly within the target with each component satisfying the equation

$$\iiint J_{2\alpha}(\vec{r}')dV' = 0 \qquad \alpha = x, y, z \tag{1}$$

We use the following notation: a prime symbol, such as *r*', refers to a source coordinate while the unprimed coordinate, such as *r*, refers to a location at which the electromagnetic field is computed. The subscripts 1 and 2 refer, respectively, to sources located in the air and in the ground. Using Tai's dyadic Green's function notation, the relationships between the electric fields and currents are

$$\vec{E}_2(\vec{r}_2) = i\omega\mu_0 \int_{V_A'} \vec{\vec{G}}^{(21)}(\vec{r}_2|\vec{r}_1') \bullet \vec{J}_1(\vec{r}_1')dV_1' \tag{2a}$$

$$\vec{E}_1(\vec{r}_1) = i\omega\mu_0 \int_{V_2'} \vec{\vec{G}}^{(12)}(\vec{r}_1|\vec{r}_2') \bullet \vec{J}_2(\vec{r}_2')dV_2' \tag{2b}$$

The dyadic Green's function $\vec{\vec{G}}^{(21)}(\vec{r}_2|\vec{r}'_1)$ connects the electric field below the earth's surface to the source above the earth's surface while $\vec{\vec{G}}^{(12)}(\vec{r}_1|\vec{r}_2')$ connects the field above the earth's surface to the ground current. The "$\bullet$" symbol is the usual dot product. Depending on the properties of $\vec{J}_2(\vec{r}_2')$ it may be more efficient to transform the right-hand-side of Equation (2b) into the following form derived from the reciprocity principle (the "~" sign means transpose).

$$\vec{\vec{G}}^{(12)}(\vec{r}_1|\vec{r}_2') = \tilde{\vec{\vec{G}}}^{(21)}(\vec{r}_2'|\vec{r}_1) \tag{3}$$

$$\begin{aligned}\vec{\vec{G}}^{(12)}(\vec{r}_1|\vec{r}_2') \bullet \vec{J}_2(\vec{r}_2') &= \tilde{\vec{\vec{G}}}^{(21)}(\vec{r}_2'|\vec{r}_1) \bullet \vec{J}_2(\vec{r}_2') = \\ &\vec{J}_2(\vec{r}_2') \bullet \tilde{\vec{\vec{G}}}^{(21)}(\vec{r}_2'|\vec{r}_1)\end{aligned} \tag{4}$$

The foregoing reciprocity results are easily derived for our case where $\mu_1 = \mu_2 = \mu_0$.

A major building block for solving the two-region system of Fig. 1 is to use the free space dyadic Green's function given by

$$\vec{\vec{G}}_0(\vec{r}\,|\,\vec{r}') = \left(\vec{\vec{I}} + (1/k^2)\nabla\nabla\right)G_0(\vec{r}\,|\,\vec{r}') \tag{5}$$

In Equation (5) the dyadic, $\vec{\vec{I}}$, is the *idem factor* ($\vec{a}_x, \vec{a}_y, \vec{a}_z$ are unit vectors), and $G_0(\vec{r}\,|\,\vec{r}')$ is the free space Green's function.

$$\vec{\vec{I}} = \vec{a}_x\vec{a}_x + \vec{a}_y\vec{a}_y + \vec{a}_z\vec{a}_z \tag{6}$$

$$G_0(\vec{r}\,|\,\vec{r}') = \exp ik\,|\vec{r}-\vec{r}'|/4\pi\,|\vec{r}-\vec{r}'| \tag{7}$$

In summary, this paper shows how to derive practical expressions for the aboveground field from target currents, Equation (2b), from the mathematical properties of the dyadic Green's function and the asymptotic approximations to complex integrals.

3. SUMMARY AND REVIEW OF DYADIC GREEN'S FUNCTION APPROACH

In this section we summarize the dyadic Green's approach for determining the electromagnetic field from

sources located above and below the ground, respectively. This material is extracted from the excellent text by Tai and is provided to allow the reader to apply the method to related scenarios. Determining the electromagnetic fields for Fig. 1 is best achieved in cylindrical coordinates.

The starting point is to determine the free-space dyadic in cylindrical geometry from the equation

$$\nabla\times\nabla\times\bar{\bar{G}}_0(\vec{r}\,|\,\vec{r}\,')-k^2\bar{\bar{G}}_0(\vec{r}\,|\,\vec{r}\,')=\bar{\bar{I}}\delta(\vec{r}-\vec{r}\,') \tag{8}$$

where $\bar{\bar{I}}\delta(\vec{r}-\vec{r}\,')$ is expressed in the following orthogonal set of basis cylindrical wave vector functions:

$$\vec{M}_{en\lambda}=\left[-\frac{nJ_n(\lambda r)}{r}\sin n\phi\,\vec{a}_r-\frac{\partial J_n(\lambda r)}{\partial r}\cos n\phi\,\vec{a}_\varphi\right]\exp(ihz) \tag{9a}$$

$$\vec{M}_{on\lambda}=\left[+\frac{nJ_n(\lambda r)}{r}\cos n\phi\,\vec{a}_r-\frac{\partial J_n(\lambda r)}{\partial r}\sin n\phi\,\vec{a}_\phi\right]\exp(ihz) \tag{9b}$$

$$\vec{N}_{en\lambda}=\frac{1}{k_\lambda}\begin{bmatrix} ih\dfrac{\partial J_n(\lambda r)}{\partial r}\cos n\phi\,\vec{a}_r-\dfrac{ihn}{r}J_n(\lambda r)\sin n\phi\,\vec{a}_\phi \\ +\lambda^2 J_n(\lambda r)\cos n\phi\,\vec{a}_z \end{bmatrix}\exp(ihz) \tag{9c}$$

$$\vec{N}_{on\lambda}=\frac{1}{k_\lambda}\begin{bmatrix} ih\dfrac{\partial J_n(\lambda r)}{\partial r}\sin n\phi\,\vec{a}_r+\dfrac{ihn}{r}J_n(\lambda r)\cos n\phi\,\vec{a}_\phi \\ +\lambda^2 J_n(\lambda r)\sin n\phi\,\vec{a}_z \end{bmatrix}\exp(ihz) \tag{9d}$$

$$k_\lambda^2=\lambda^2+h^2 \tag{9e}$$

In the foregoing equations $J_n(\lambda r)$ is an integer Bessel function of order *n*.

Equations for the two-region problem of Fig. 1 are

$$\nabla\times\nabla\times\vec{E}_1-k_1^2\vec{E}_1=i\omega\mu_0\vec{J}_1 \qquad z\ge 0 \tag{10}$$

$$\nabla\times\nabla\times\vec{E}_2-k_1^2\vec{E}_2=0 \qquad z<0 \tag{11}$$

At the ground-air interface we have

$$\vec{n}\times\vec{E}_1=\vec{n}\times\vec{E}_2 \tag{12}$$

$$\vec{n}\times\vec{H}_1=\vec{n}\times\vec{H}_2 \tag{13}$$

$$\nabla\times\vec{E}=i\omega\mu_0\vec{H} \tag{14}$$

$$\nabla\times\vec{H}=-i\omega\varepsilon\vec{E} \tag{15}$$

$$\frac{\vec{n}\times\nabla\times\vec{H}_1}{-i\omega\varepsilon_1}=\frac{\vec{n}\times\nabla\times\vec{H}_2}{-i\omega\varepsilon_2} \tag{16}$$

$$\vec{n}\times\nabla\times\vec{E}_1=\vec{n}\times\nabla\times\vec{E}_2 \tag{17}$$

When the source lies above the ground, we use the following Green's functions:

$$\nabla\times\nabla\times\bar{\bar{G}}_3^{(11)}-k_1^2\bar{\bar{G}}_3^{(11)}=\bar{\bar{I}}\delta(\vec{r}-\vec{r}\,') \qquad z\ge 0 \tag{18}$$

$$\nabla\times\nabla\times\bar{\bar{G}}_3^{(21)}-k_2^2\bar{\bar{G}}_3^{(21)}=0 \qquad z\ge 0 \tag{19}$$

If the source is in the lower region, we have

$$\nabla\times\nabla\times\bar{\bar{G}}_3^{(12)}-k_1^2\bar{\bar{G}}_3^{(12)}=0 \qquad z<0 \tag{20}$$

$$\nabla\times\nabla\times\bar{\bar{G}}_3^{(22)}-k_2^2\bar{\bar{G}}_3^{(22)}=\bar{\bar{I}}\delta(\vec{r}-\vec{r}\,') \qquad z<0 \tag{21}$$

For the $z\ge 0$ case we have

$$\vec{E}_1(\vec{r}\,)=i\omega\mu_0\int_{V_1'}\bar{\bar{G}}_3^{(11)}(\vec{r}\,\big|\,\vec{r}\,')\bullet\vec{J}_1(\vec{r}\,')dV_1' \tag{22}$$

$$\vec{E}_2(\vec{r}\,)=i\omega\mu_0\int_{V_1'}\bar{\bar{G}}_3^{(21)}(\vec{r}\,\big|\,\vec{r}\,')\bullet\vec{J}_1(\vec{r}\,')dV_1' \tag{23}$$

For the $z<0$ case we have

$$\vec{E}_1(\vec{r}\,)=i\omega\mu_0\int_{V_2'}\bar{\bar{G}}_3^{(12)}(\vec{r}\,\big|\,\vec{r}\,')\bullet\vec{J}_2(\vec{r}\,')dV_2' \tag{24}$$

$$\vec{E}_2(\vec{r}\,)=i\omega\mu_0\int_{V_A'}\bar{\bar{G}}_3^{(22)}(\vec{r}\,\big|\,\vec{r}\,')\bullet\vec{J}_2(\vec{r}\,')dV_2' \tag{25}$$

From the relationship $\bar{\bar{D}}\bullet\vec{C}=\vec{C}\bullet\tilde{\bar{\bar{D}}}$ we get the equivalent ways of computing $\vec{E}_1(\vec{r}\,)$ and $\vec{E}_2(\vec{r}\,)$:

$$\vec{E}_1(\vec{r}\,)=i\omega\mu_0\int_{V_2'}\vec{J}_1(\vec{r}\,')\bullet\tilde{\bar{\bar{G}}}_3^{(11)}(\vec{r}\,\big|\,\vec{r}\,')dV\,' \tag{26}$$

$$\vec{E}_1(\vec{r}\,)=i\omega\mu_0\int_{V_2'}\vec{J}_1(\vec{r}\,')\bullet\bar{\bar{G}}_3^{(11)}(\vec{r}\,'\big|\,\vec{r}\,)dV\,' \tag{27}$$

$$\vec{E}_2(\vec{r}\,)=i\omega\mu_0\int_{V_2'}\vec{J}_1(\vec{r}\,')\bullet\tilde{\bar{\bar{G}}}_3^{(21)}(\vec{r}\,\big|\,\vec{r}\,')dV\,' \tag{28}$$

$$\vec{E}_2(\vec{r}\,)=i\omega\mu_0\int_{V_2'}\vec{J}_1(\vec{r}\,')\bullet\bar{\bar{G}}_3^{(21)}(\vec{r}\,'\big|\,\vec{r}\,)dV\,' \tag{29}$$

The building blocks for the solution of the different Green's functions are the wave vector functions used in conjunction with $\vec{\vec{G}}_0(\vec{r}\,|\,\vec{r}\,')$.

In region 1 with $z, z' \geq 0$ and $z \geq z'$, we have:

$$\vec{\vec{G}}_0(\vec{r}\,|\,\vec{r}\,') = \frac{i}{4\pi}\int_0^\infty d\lambda \sum_{n=0}^\infty \frac{2-\delta_0}{\lambda h_1}\begin{bmatrix} \vec{M}_{en\lambda}(h_1)\vec{M}'_{en\lambda}(-h_1) + \\ \vec{M}_{on\lambda}(h_1)\vec{M}'_{on\lambda}(-h_1) + \\ \vec{N}_{en\lambda}(h_1)\vec{N}'_{en\lambda}(-h_1) + \\ \vec{N}_{on\lambda}(h_1)\vec{N}'_{on\lambda}(-h_1) \end{bmatrix} \tag{30}$$

In region 1 with $z, z' \geq 0$ and $z' \geq z$, we have

$$\vec{\vec{G}}_0(\vec{r}\,|\,\vec{r}\,') = \frac{i}{4\pi}\int_0^\infty d\lambda \sum_{n=0}^\infty \frac{2-\delta_0}{\lambda h_1}\begin{bmatrix} \vec{M}_{en\lambda}(-h_1)\vec{M}'_{en\lambda}(h_1) + \\ \vec{M}_{on\lambda}(-h_1)\vec{M}'_{on\lambda}(h_1) + \\ \vec{N}_{en\lambda}(-h_1)\vec{N}'_{en\lambda}(h_1) + \\ \vec{N}_{on\lambda}(-h_1)\vec{N}'_{on\lambda}(h_1) \end{bmatrix} \tag{31}$$

The prime notation used in Equations (30) and (31), such as $\vec{M}'_{en\lambda}$, means that it is a function of the primed coordinates, r', ϕ', z'. The following definitions apply:

$$\delta_0 = 1 \quad ; for\ m\ or\ n = 0 \tag{32a}$$
$$\delta_0 = 0 \quad ; for\ m\ or\ n \neq 0 \tag{32b}$$

$$k_1 = \omega\sqrt{\mu_0\varepsilon_0} \tag{33}$$

$$k_2 = \omega\sqrt{\mu_0\varepsilon(1 + i\sigma/\omega\varepsilon)} \tag{34}$$

$$h_1 = \sqrt{k_1^2 - \lambda^2} \tag{35}$$

$$h_2 = \sqrt{k_2^2 - \lambda^2} \tag{36}$$

To solve Equations (18) and (19) we assume

$$\vec{\vec{G}}_3^{(11)}(\vec{r}\,|\,\vec{r}\,') = \vec{\vec{G}}_0(\vec{r}\,|\,\vec{r}\,') + \vec{\vec{G}}_{3s}^{(11)}(\vec{r}\,|\,\vec{r}\,') \qquad z \geq 0 \tag{37}$$

$$\vec{\vec{G}}_3^{(21)}(\vec{r}\,|\,\vec{r}\,') = \vec{\vec{G}}_{3s}^{(21)}(\vec{r}\,|\,\vec{r}\,') \qquad z \leq 0 \tag{38}$$

In order to satisfy the boundary conditions at the interface between the two regions and the radiation conditions at $z = \pm\infty$, we must have

$$\vec{\vec{G}}_{3s}^{(11)}(\vec{r}\,|\,\vec{r}\,') = \frac{i}{4\pi}\int_0^\infty d\lambda \sum_{n=0}^\infty \frac{2-\delta_0}{\lambda h_1}\begin{bmatrix} a\vec{M}_{en\lambda}(h_1)\vec{M}'_{en\lambda}(h_1) + \\ a\vec{M}_{on\lambda}(h_1)\vec{M}'_{on\lambda}(h_1) + \\ b\vec{N}_{en\lambda}(h_1)\vec{N}'_{en\lambda}(h_1) + \\ b\vec{N}_{on\lambda}(h_1)\vec{N}'_{on\lambda}(h_1) \end{bmatrix} \tag{39}$$

$$\vec{\vec{G}}_{3s}^{(21)}(\vec{r}\,|\,\vec{r}\,') = \frac{i}{4\pi}\int_0^\infty d\lambda \sum_{n=0}^\infty \frac{2-\delta_0}{\lambda h_1}\begin{bmatrix} c\vec{M}_{en\lambda}(-h_2)\vec{M}'_{en\lambda}(h_1) + \\ c\vec{M}_{on\lambda}(-h_2)\vec{M}'_{on\lambda}(h_1) + \\ d\vec{N}_{en\lambda}(-h_2)\vec{N}'_{en\lambda}(h_1) + \\ d\vec{N}_{on\lambda}(-h_2)\vec{N}'_{on\lambda}(h_1) \end{bmatrix} \tag{40}$$

The unknown constants: *a*, *b*, *c*, and *d* are found by matching the boundary conditions at the interface, $z = 0$. We have

$$\vec{n}\times\vec{\vec{G}}_3^{(11)} = \vec{n}\times\vec{\vec{G}}_3^{(21)} \tag{41}$$

$$\vec{n}\times\nabla\times\vec{\vec{G}}_3^{(11)} = \vec{n}\times\nabla\times\vec{\vec{G}}_3^{(21)} \tag{42}$$

$$a = \frac{h_1 - h_2}{h_1 + h_2} \tag{43}$$

$$b = \frac{k_2^2 h_1 - k_1^2 h_2}{k_2^2 h_1 + k_1^2 h_2} \tag{44}$$

$$c = \frac{2h_1}{h_1 + h_2} \tag{45}$$

$$d = \frac{2k_1 k_2 h_1}{k_2^2 h_1 + k_1^2 h_2} \tag{46}$$

4. ELECTROMAGNETIC FIELD DETECTION

The model described by Equation (1) assumes no net current flowing into or out of the source volume, *V'*. All current circulates within the volume. For brevity we assume no current flow in the *z* direction. We then have

$$\iint J_{2x}(x', y')dx'dy' = 0 \tag{47}$$

$$\iint J_{2y}(x', y')dx'dy' = 0 \tag{48}$$

Even though the foregoing equations are satisfied, we still have $J_x(x', y') \neq 0$, $J_y(x', y') \neq 0$ for $x', y' \in V'$. Using the

relationships $x'=r'\cos\phi'$ and $y'=r'\sin\phi'$ we write the radial and angular components of current density as

$$J_{2r}(r',\phi')=\cos\phi' J_{2x}(r',\phi')+\sin\phi' J_{2y}(r',\phi') \tag{49}$$

$$J_{2\phi}(r',\phi')=\cos\phi' J_{2x}(r',\phi')+\sin\phi' J_{2y}(r',\phi') \tag{50}$$

Using the unit vectors $\vec{a}_r,\vec{a}_\phi$ the vector current density is

$$\vec{J}_2(r',\phi')=J_{2r}(r',\phi')\vec{a}_r+J_{2\varphi}(r',\phi')\vec{a}_\varphi \tag{51}$$

The aboveground field is computed from the source term below ground.

$$\vec{E}_1(\vec{r}_1)=i\omega\mu_0\int_{V_2'}\vec{\vec{G}}_3^{(12)}(\vec{r}_1|\vec{r}_2')\bullet\vec{J}_2(\vec{r}_2')dV_2' \tag{52}$$

It is not necessary to determine $\vec{\vec{G}}_3^{(12)}(\vec{r}_1|\vec{r}_2')$ by repeating the calculations of the previous section. All that is necessary is to interchange the roles of k_1 and k_2, and the vertical positions of the transmitter and target, z_1, z_2, respectively. Thus, all the previous formulas are applicable with the following substitutions (new values are indicated with the "hat" symbol such as $\hat{a}$):

$$\hat{k}_2=\omega\sqrt{\mu_0\varepsilon_0}=k_1 \tag{53}$$

$$\hat{k}_1=\omega\sqrt{\mu_0\varepsilon(1+i\sigma/\omega\varepsilon)}=k_2 \tag{54}$$

$$\hat{h}_2=\sqrt{\hat{k}_2^2-\lambda^2}=h_1 \tag{55}$$

$$\hat{h}_1=\sqrt{\hat{k}_1^2-\lambda^2}=h_2 \tag{56}$$

$$\hat{a}=\frac{\hat{h}_1-\hat{h}_2}{\hat{h}_1+\hat{h}_2}=-a \tag{57}$$

$$\hat{b}=\frac{\hat{k}_2^2\hat{h}_1-\hat{k}_1^2\hat{h}_2}{\hat{k}_2^2\hat{h}_1+\hat{k}_1^2\hat{h}_2}=-b \tag{58}$$

$$\hat{c}=\frac{2\hat{h}_1}{\hat{h}_1+\hat{h}_2}=\frac{2h_2}{h_1+h_2} \tag{59}$$

$$\hat{d}=\frac{2\hat{k}_1\hat{k}_2\hat{h}_1}{\hat{k}_2^2\hat{h}_1+\hat{k}_1^2\hat{h}_2}=\frac{2k_1k_2h_2}{k_1^2h_2+k_2^2h_1} \tag{60}$$

$$\hat{z}_1=z_2 \tag{61}$$

$$\hat{z}_2=z_1 \tag{62}$$

With this upside-down frame of reference, we replace $\vec{\vec{G}}_3^{(12)}(\vec{r}\,|\,\vec{r}')$ by $\vec{\vec{G}}_{3s}^{(21)}(\vec{r}\,|\,\vec{r}')$, and we let r,ϕ denote the field point of interest and r',ϕ' the source coordinates. To minimize and simplify the notation, it is now stated that the field point is above the ground (we drop the subscript "1") and the current is the target current (we drop the subscript "2"). We still retain the subscripts for the parameters defined by Equations (53) to (62). Since the height of the true field point is the positive z_1, it is now the below-surface fictitious depth, $-z_1$, and positive z_2 is now the height of the source. With this new notation Equation (52) now reads

$$\vec{E}(\vec{r})=i\omega\mu_0\int_{V'}\vec{\vec{G}}_3^{(21)}(\vec{r}\,|\vec{r}\,')\bullet\vec{J}(\vec{r}\,')dV' \tag{63}$$

We have

$$i\omega\mu_0\vec{\vec{G}}_{3s}^{(21)}=-\int_0^\infty d\lambda\sum_{n=0}^{\infty}\frac{\omega\mu_0(2-\delta_0)}{4\pi\lambda\hat{h}_1}\begin{bmatrix}\hat{c}\vec{M}_{en\lambda}(-\hat{h}_2)\vec{M}'_{en\lambda}(\hat{h}_1)+\\ \hat{c}\vec{M}_{on\lambda}(-\hat{h}_2)\vec{M}'_{on\lambda}(\hat{h}_1)+\\ \hat{d}\vec{N}_{en\lambda}(-\hat{h}_2)\vec{N}'_{en\lambda}(\hat{h}_1)+\\ \hat{d}\vec{N}_{on\lambda}(-\hat{h}_2)\vec{N}'_{on\lambda}(\hat{h}_1)\end{bmatrix}$$

(63)

$$i\omega\mu_0\vec{\vec{G}}_{3s}^{(21)}=-\int_0^\infty d\lambda\sum_{n=0}^{\infty}\frac{\omega\mu_0(2-\delta_0)}{4\pi\lambda h_2}\begin{bmatrix}\hat{c}\vec{M}_{en\lambda}(-h_1)\vec{M}'_{en\lambda}(h_2)+\\ \hat{c}\vec{M}_{on\lambda}(-h_1)\vec{M}'_{on\lambda}(h_2)+\\ \hat{d}\vec{N}_{en\lambda}(-h_1)\vec{N}'_{en\lambda}(h_2)+\\ \hat{d}\vec{N}_{on\lambda}(-h_1)\vec{N}'_{on\lambda}(h_2)\end{bmatrix}$$

(64)

$$i\omega\mu_0\vec{\vec{G}}_{3s}^{(21)}(\vec{r}\,|\,\vec{r}')=i\omega\mu_0\sum_{n=0}^{n=\infty}\vec{\vec{G}}_n \tag{65}$$

$$\vec{E}(\vec{r})=\sum_{n=0}^{n=\infty}\int_{V'}i\omega\mu_0\vec{\vec{G}}_n\bullet\vec{J}(\vec{r}')dV' \tag{66}$$

$$\vec{E}(\vec{r})=\sum_{n=0}^{n=\infty}\vec{E}_n(\vec{r}) \tag{67}$$

$$i\omega\mu_0\vec{\vec{G}}_n=-\int_0^\infty d\lambda\frac{\omega\mu_0(2-\delta_0)}{4\pi\lambda h_2}\left[\vec{\vec{\Psi}}_{An}+\vec{\vec{\Psi}}_{Bn}+\vec{\vec{\Psi}}_{Cn}+\vec{\vec{\Psi}}_{Dn}\right] \tag{68}$$

$$\vec{\vec{\Psi}}_{An}=\hat{c}\vec{M}_{en\lambda}(-h_1)\vec{M}'_{en\lambda}(h_2) \tag{69a}$$

$$\vec{\vec{\Psi}}_{Bn} = \hat{c}\vec{M}_{on\lambda}(-h_1)\vec{M}'_{on\lambda}(h_2) \quad (69b)$$

$$\vec{\vec{\Psi}}_{Cn} = \hat{d}\vec{M}_{en\lambda}(-h_1)\vec{M}'_{en\lambda}(h_2) \quad (69c)$$

$$\vec{\vec{\Psi}}_{Dn} = \vec{d}\vec{M}_{on\lambda}(-h_1)\vec{M}'_{on\lambda}(h_2) \quad (69d)$$

$$\vec{E}_n(\vec{r}) = \vec{E}_{An}(\vec{r}) + \vec{E}_{Bn}(\vec{r}) + \vec{E}_{Cn}(\vec{r}) + \vec{E}_{Dn}(\vec{r}) \quad (70)$$

$$\vec{E}_{An}(\vec{r}) = -\int_0^\infty \frac{\omega\mu_0(2-\delta_0)}{4\pi\lambda h_2} d\lambda \int_{V'} \vec{\vec{\Psi}}_{An}(\vec{r}\,|\,\vec{r}\,') \bullet \vec{J}(\vec{r}\,')dV' \quad (71a)$$

$$\vec{E}_{Bn}(\vec{r}) = -\int_0^\infty \frac{\omega\mu_0(2-\delta_0)}{4\pi\lambda h_2} d\lambda \int_{V'} \vec{\vec{\Psi}}_{Bn}(\vec{r}\,|\,\vec{r}\,') \bullet \vec{J}(\vec{r}\,')dV' \quad (71b)$$

$$\vec{E}_{Cn}(\vec{r}) = -\int_0^\infty \frac{\omega\mu_0(2-\delta_0)}{4\pi\lambda h_2} d\lambda \int_{V'} \vec{\vec{\Psi}}_{Cn}(\vec{r}\,|\,\vec{r}\,') \bullet \vec{J}(\vec{r}\,')dV' \quad (71c)$$

$$\vec{E}_{Dn}(\vec{r}) = -\int_0^\infty \frac{\omega\mu_0(2-\delta_0)}{4\pi\lambda h_2} d\lambda \int_{V'} \vec{\vec{\Psi}}_{Dn}(\vec{r}\,|\,\vec{r}\,') \bullet \vec{J}(\vec{r}\,')dV' \quad (71d)$$

By example, we carry out the calculation for $\vec{E}_{An}(\vec{r})$.

$$\int_{V'} \vec{\vec{\Psi}}_{An}(\vec{r}\,|\,\vec{r}\,') \bullet \vec{J}(\vec{r}\,')dV' = \hat{c}\vec{M}_{en\lambda}(-h_1)\int_{V'} \vec{M}'_{en\lambda}(h_2) \bullet \vec{J}(\vec{r}\,')dV' \quad (72)$$

$$\vec{M}'_{en\lambda}(h_2) \bullet \vec{J}(\vec{r}\,') = \exp(ih_2 z_2) \left[-\frac{nJ_n(\lambda r')}{r'}\sin n\phi'\vec{a}_r - \frac{\partial J_n(\lambda r')}{\partial r'}\cos n\phi'\vec{a}_\varphi\right] \bullet [J_r\vec{a}_r + J_\phi\vec{a}_\phi] \quad (73)$$

The terms denoted by $J_n(\lambda r')$ again refer to Bessel functions of integer order "*n*", and are not to be confused with the components of current J_r and J_ϕ.

Integrating Equation (73) over dV' gives

$$\int_{V'} \vec{M}'_{en\lambda}(h_2) \bullet \vec{J}(\vec{r}\,')dV' = \exp(ih_2 z_2)\Gamma_{An} \quad (74)$$

$$\Gamma_{An} = \Gamma_{An}^{(1)} + \Gamma_{An}^{(2)} \quad (75a)$$

$$\Gamma_{An}^{(1)} = -\int_{V'} \frac{nJ_n(\lambda r')}{r'}\sin n\phi' J_r dV' \quad (75b)$$

$$\Gamma_{An}^{(2)} = -\int_{V'} \frac{\partial J_n(\lambda r')}{\partial r'}\cos n\phi' J_\phi dV' \quad (75c)$$

Since there is no vertical component of volume to consider (in this case J_r and J_ϕ have the dimensions of amps per unit meter), we have

$$dV' = r'dr'd\varphi' \quad (76)$$

$$\Gamma_{An}^{(1)} = -\int_0^L\int_0^{2\pi} \frac{nJ_n(\lambda r')}{r'}\sin n\phi' J_r r'dr'd\phi' \quad (77a)$$

$$\Gamma_{An}^{(2)} = -\int_0^L\int_0^{2\pi} \frac{\partial J_n(\lambda r')}{\partial r'}\cos n\phi' J_\phi r'dr'd\phi' \quad (77b)$$

The components, $J_r(r',\phi')$ and $J_r(r',\phi')$ are confined in the horizontal plane to a small area bounded by a circle with radius, $r'_{\max} = L << 1/k_1$. In this case we use the expansion

$$J_r(r',\phi') = \sum_{l=0}^{l=\infty} P_l(r')\sin l\phi' + \sum_{l=0}^{l=\infty} Q_l(r')\cos l\phi' \quad (78a)$$

$$J_\phi(r',\phi') = \sum_{l=0}^{l=\infty} R_l(r')\sin l\phi' + \sum_{l=0}^{l=\infty} S_l(r')\cos l\phi' \quad (78b)$$

The radial functions of Equation (78) are analytic with no singularities. Using the formulas

$$\int_0^{2\pi} \sin^2 n\phi' d\varphi' = \int_0^{2\pi} \cos^2 n\phi' d\varphi' = \pi \quad (79a)$$

$$\int_0^{2\pi} \sin n\phi' \cos m\phi' d\varphi' = 0 \quad (79b)$$

we get

$$\Gamma_{An}^{(1)}(\lambda) = -\pi\int_0^L nJ_n(\lambda r')P_n(r')dr' \quad (80a)$$

$$\Gamma_{An}^{(2)}(\lambda) = -\pi\int_0^L \frac{\partial J_n(\lambda r')}{\partial r'} S_n(r')r'dr' \quad (80b)$$

$$\Gamma_{An}(\lambda) = \Gamma_{An}^{(1)}(\lambda) + \Gamma_{An}^{(1)}(\lambda) \quad (80c)$$

Inserting the foregoing results into Equation (71a) gives

$$\vec{E}_{An}(\vec{r}) = \int_0^\infty \frac{\omega\mu_0(2-\delta_0)}{4\pi\lambda h_2}\hat{c}\vec{M}_{en\lambda}(-h_1)\exp(ih_2 z_2)\Gamma_{An} d\lambda \quad (81)$$

$$\frac{\hat{c}}{h_2} = \frac{2}{h_1 + h_2} \tag{82}$$

$$\vec{M}_{en\lambda}(-h_1)\exp(ih_2 z_2) = -\left[\begin{array}{l} \dfrac{nJ_n(\lambda r)}{r}\sin n\phi\,\vec{a}_r + \\ \dfrac{\partial J_n(\lambda r)}{\partial r}\cos n\phi\,\vec{a}_\varphi \end{array}\right]$$

$$\exp(ih_1 z_1 + ih_2 z_2) \tag{83}$$

$$\vec{E}_{An}(\vec{r}\,) = -\frac{\omega\mu_0(2-\delta_0)}{4\pi}K_n\vec{a}_r - \frac{\omega\mu_0(2-\delta_0)}{4\pi}L_n\vec{a}_\phi \tag{84}$$

$$K_n = \int_0^\infty \left[\begin{array}{l} \dfrac{2}{\lambda(h_1+h_2)}\exp(ih_1z_1 + ih_2z_2)\dfrac{nJ_n(\lambda r)}{r} \\ \Gamma_{An}\sin n\phi \end{array}\right] d\lambda \tag{85}$$

$$L_n = \int_0^\infty \left[\begin{array}{l} \dfrac{2}{\lambda(h_1+h_2)}\exp(ih_1z_1 + ih_2z_2)\dfrac{\partial J_n(\lambda r)}{\partial r} \\ \Gamma_{An}\cos n\phi \end{array}\right] d\lambda \tag{86}$$

Following the procedure leading to Equations (84) to (86), we readily compute the remaining terms of Equation (70) and thereby compute the aboveground field to be detected. In the next section we render a sample calculation.

5. COMPUTATIONAL CONSIDERATIONS AND SAMPLE CALCULATION

In this section we examine the challenges in computing the integrals in Equations (84) to (86), and then render a sample calculation. We first cast Equations (84) to (86) in the form

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$$\vec{E}_{An}(\vec{r}\,) = -\frac{\omega\mu_0(2-\delta_0)n\sin n\phi}{2\pi r}\overline{K}_n\vec{a}_r - \frac{\omega\mu_0(2-\delta_0)\cos n\phi}{2\pi}\overline{L}_n\vec{a}_\phi \tag{87}$$

$$\overline{K}_n = \int_0^\infty \left[\frac{J_n(\lambda r)\Gamma_{An}}{\lambda(h_1+h_2)}\exp(ih_1z_1 + ih_2z_2)\right] d\lambda \tag{88}$$

$$\overline{L}_n = \int_0^\infty \left[\frac{\Gamma_{An}}{\lambda(h_1+h_2)}\frac{\partial J_n(\lambda r)}{\partial r}\exp(ih_1z_1 + ih_2z_2)\right] d\lambda \tag{89}$$

The foregoing equations are in a suitable form for numerical integration provided we are aware of the guiding principle that the sum of all constituents of the field, as given by Equation (70), must satisfy the radiation conditions at $z_1 \to \infty$ and $z_2 \to \infty$. That is, the electromagnetic field must go to zero at these limits. Concern about this arises from the two branch points connected with $h_1 = \sqrt{k_1^2 - \lambda^2}$ and $h_2 = \sqrt{k_2^2 - \lambda^2}$ because of the ambiguity of taking the + or – square root. These branch points each then lead to two branch cuts, and each branch cut leads to two Riemann surfaces. Thus, we have a total of four Riemann surfaces to consider. At every step of the numerical calculation the radiation condition must be satisfied.

We note in passing that even though there is a "λ^{-1}"-factor in both integrands of Equations (85) and (86), this does not lead to a singularity or a pole. In the limit of $\lambda \to 0$ we have $J_n(\lambda r) \to \lambda^n$, so that the only term that could yield λ^{-1} in this domain is $J_0(\lambda r)$. But as we see from Equation (85), the factor "n" ensures that this whole term vanishes for $n = 0$. We also have

$$\frac{\partial J_n(\lambda r)}{\partial r} = \lambda\frac{\partial J_n(\varsigma)}{\partial \varsigma} \tag{90}$$

where $\varsigma = \lambda r$. Since $\dfrac{\partial J_n(\varsigma)}{\partial\varsigma}$ is always finite, the λ term from Equation (90) cancels λ^{-1} and the apparent pole or singularity disappears.

We have not yet attempted a numerical evaluation of Equations (88) and (89). Such a computation will be considered in the event that a near-term analytical approach based on the "Method of Steepest Descent" (MSD) is not fruitful. The mathematical structure of the integrands of Equations (88) and (89) is consistent with that used in MSD (Chew, 1990) and in the Ohio State mathematics lecture series. Let S represent either $\overline{K}_n$ or $\overline{L}_n$ since they are equivalent in structure, $Q(\lambda)$ represent either $\dfrac{J_n(\lambda r)\Gamma_{An}}{\lambda(h_1+h_2)}$ or $\dfrac{\Gamma_{An}}{\lambda(h_1+h_2)}\dfrac{\partial J_n(\lambda r)}{\partial r}$, and

$$f(\lambda) = ih_1z_1 + ih_2z_2. \tag{91}$$

The MSD form of Equations (88) and (89) is

$$S = \int_0^\infty Q(\lambda)\exp f(\lambda)\,d\lambda \tag{92}$$

Branch points from $h_1(\lambda)$ and $h_2(\lambda)$ require integration in the complex plane using $\lambda = x + iy$. Asymptotic approximations to Equation (92) require a number of conditions. First, we must have the situation where $Q(\lambda)$ is a slowly varying function of λ in the region where

$\exp f(\lambda)$ is the major contributor to the integral. Next, we require that $f(\lambda) = u(x,y) + iv(x,y)$ be analytic and the functions $u(x,y)$ and $v(x,y)$ satisfy the Cauchy-Riemann equations. Assume that a saddle-point extremum of $f(\lambda)$ exists at a point λ_0 determined by solving the equivalent equations

$$\left(\frac{\partial f}{\partial x}\right)_{\lambda_0} = 0 = \left(\frac{\partial f}{\partial y}\right)_{\lambda_0} \tag{93a}$$

From the Cauchy-Riemann equations we can also write

$$\left(\frac{df}{d\lambda}\right)_{\lambda_0} = 0 \tag{93b}$$

Using Equation (93) we have the following result in the neighborhood around λ_0

$$f(\lambda) = f(\lambda_0) + \frac{1}{2}(\lambda - \lambda_0)^2 \left(\frac{d^2 f}{d\lambda^2}\right)_{\lambda_0} + \ldots \tag{94}$$

Using the foregoing equation combined with other mathematical steps, we get the following asymptotic approximation to Equation (92) based on higher-order terms not shown in Equation (94), taken from Ohio State math lecture notes.

$$S = \frac{\sqrt{2\pi}\,\exp f(\lambda_0)}{\left[-\left(\dfrac{d^2 f}{d\lambda^2}\right)_{\lambda_0}\right]^{1/2}} \sum_{m=0} \left(\frac{d^{2m} Q}{d\lambda^{2m}}\right)_{\lambda_0} \frac{(-1)^m}{2^m\, m!} \left(\frac{1}{\left(\dfrac{d^2 f}{d\lambda^2}\right)_{\lambda_0}}\right)^m \tag{95}$$

For illustrative purposes we apply the method for a canonical form such as $f(\lambda) = ih_1 z_1 + ih_2 z_2$ with z_1 positive and z_2 negative. Using Equation (93b) for $h_1 = \sqrt{k_1^2 - \lambda^2}$ and $h_2 = \sqrt{k_2^2 - \lambda^2}$ we readily get

$$\lambda_0^2 = \frac{k_2^2 - r k_1^2}{1 - r} \tag{96}$$

$$r = \frac{z_2^2}{z_1^2} \tag{97}$$

From λ_0 we straightforwardly calculate the functions identified in Equation (95). In addition the summation in Equation (95) provides higher-order corrections.

CONCLUSIONS

Using the dyadic Green's function approach this study provides a rigorous methodology for determining a detectible aboveground electromagnetic field for a class of buried target currents. Numerical computational techniques, as well as totally analytical techniques based on the asymptotic theory of integration, are examined.

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